Convert the polar equation $r = 3 - 5\sin 2\theta$ to rectangular, and simplify as shown in the website handout. **SCORE: _____ / 5 PTS**

NOTE: Your final answer should NOT have fractions, fractional exponents nor radicals.

$$r = 3 - 5(2\sin\theta\cos\theta)$$

$$r = 3 - 10 \cdot \frac{y}{r}\frac{x}{r}$$

$$r^3 = 3r^2 - 10xy$$

$$(\sqrt{x^2+y^2})^3 = 3(x^2+y^2) - 10xy$$

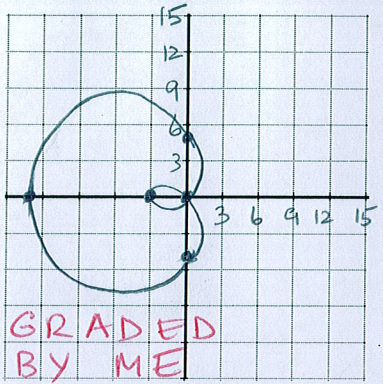
$$= 3x^2 - 10xy + 3y^2$$

$$(x^2+y^2)^3 = (3x^2 - 10xy + 3y^2)^2$$

1 POINT EACH

Consider the graph of the polar equation $r = 5 - 8\cos\theta$.

SCORE: _____ / 6 PTS



[a] Fill in the blanks.

$\left|\frac{5}{-8}\right| < 1$ MUST HAVE "LOOP" TO EARN ANY POINTS

[i] The shape of the graph is a/an LIMAÇON WITH LOOP (1).

[ii] The graph (1) DOES pass through the pole.
does / does not

[iii] Find the **rectangular coordinates** of the

θ	r
0	-3
$\frac{\pi}{2}$	5
π	13
$\frac{3\pi}{2}$	5

x – intercept(s) (½) $(-3, 0)$, $(-13, 0)$ (½) $(0, 0)$

y – intercept(s) (1) $(0, \pm 5)$ $(0, 0)$

[b] Sketch the graph on the grid provided above. **You must provide a scale for the axes & plot all points from part [a][iii] above.**

Fill in the blanks.

SCORE: _____ / 4 PTS

[a] The polar co-ordinates $(-11, -\frac{12\pi}{7})$ correspond to the same point as the polar co-ordinates $(-11, \underline{\boxed{\frac{2\pi}{7}}})$ and $(11, \underline{\boxed{\frac{9\pi}{7}}})$.

1 1

$-\frac{12\pi}{7} + 2\pi$ $\quad$ $\frac{2\pi}{7} + \pi$

NOTE: Both your answers for this question must be positive.

[b] The point with rectangular co-ordinates $(-\sqrt{6}, -3\sqrt{2})$ has polar co-ordinates $(\underline{\boxed{2\sqrt{6}}}, \underline{\boxed{\frac{4\pi}{3}}})$.

1

$r^2 = (-\sqrt{6})^2 + (-3\sqrt{2})^2$

$= 6 + 18 = 24$

$\theta = \pi + \tan^{-1}\frac{-3\sqrt{2}}{-\sqrt{6}}$

$= \pi + \tan^{-1}\sqrt{3} = \pi + \frac{\pi}{3}$

NOTE: Both parts of your answer for this question must be positive.

[c] The point with polar co-ordinates $(4, -\frac{5\pi}{6})$ has rectangular co-ordinates $(\underline{\boxed{-2\sqrt{3}}}, \underline{\boxed{-2}})$.

1

$x = 4\cos(-\frac{5\pi}{6}) = 4 \cdot -\frac{\sqrt{3}}{2}$ $\quad$ $y = 4\sin(-\frac{5\pi}{6}) = 4 \cdot -\frac{1}{2}$

Consider the graph of the polar equation $r = 3 - 6\cos 4\theta$. **SCORE: _____ / 15 PTS**

NOTE: $(-r, \theta)$ and $(-r, \pi-\theta)$ tests do NOT show that the graph is symmetric

POLE POLAR AXIS

[a] **Using the information above, and the tests and shortcuts shown in lecture**, test if the graph is symmetric over the pole, the polar axis, and/or $\theta = \frac{\pi}{2}$. State your conclusions in the table. **NOTE: Run as FEW tests as needed to prove your answers are correct.**

POLE: $r = 3 - 6\cos 4(\pi+\theta)$ (1)

$r = 3 - 6\cos(4\pi + 4\theta)$

$r = 3 - 6[\cos 4\pi \cos 4\theta - \sin 4\pi \sin 4\theta]$

$r = 3 - 6\cos 4\theta$ (1) SYM

POLAR AXIS: $r = 3 - 6\cos 4(-\theta)$ (1)

$r = 3 - 6\cos 4\theta$ (1) SYM

AUTOMATICALLY SYM OVER $\theta = \frac{\pi}{2}$

Type of symmetry	Conclusion
Over the polar axis	SYMMETRIC
Over $\theta = \frac{\pi}{2}$	SYMMETRIC
Over the pole	SYMMETRIC

(1) POINT IF EXACTLY 2 CORRECT

(2) POINTS IF ALL CORRECT

[b] Based on the results of part [a], what is the minimum interval of the graph you need to plot first (before using reflections to draw the rest of the graph) ?

$[0, \frac{\pi}{2}]$ (1)

[c] Find the angles **algebraically** in the minimum interval in part [b] at which the graph goes through the pole.

$0 = 3 - 6\cos 4\theta$ $\quad 0 \le \theta \le \frac{\pi}{2} \rightarrow 0 \le 4\theta \le 2\pi$

$6\cos 4\theta = 3$

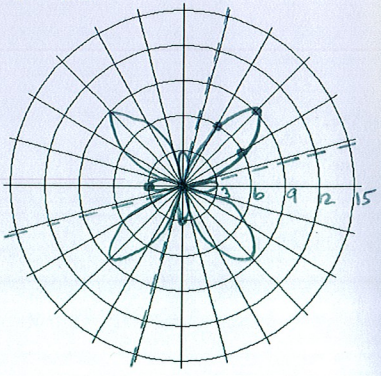
$\frac{1}{2}$ $\cos 4\theta = \frac{1}{2}$ $\quad 4\theta = \frac{\pi}{3}, \frac{5\pi}{3}$ (1)

$\theta = \frac{\pi}{12}, \frac{5\pi}{12}$ (1)

[d] Find the value of r (rounded to 1 decimal place) for all common angles in the minimum interval in part [b].
NOTE: You do NOT need to show work, only answers.

θ	r
0	-3
$\frac{\pi}{6}$	6
$\frac{\pi}{4}$	9
$\frac{\pi}{3}$	6
$\frac{\pi}{2}$	-3

$\frac{1}{2}$ POINT EACH



GRADED BY ME

[d] Sketch the graph on the grid provided below. **You must provide a scale for the polar axis & plot all points from part [c] above.**